Spreading of wave packets in one dimensional disordered chains.
I. Different dynamical regimes

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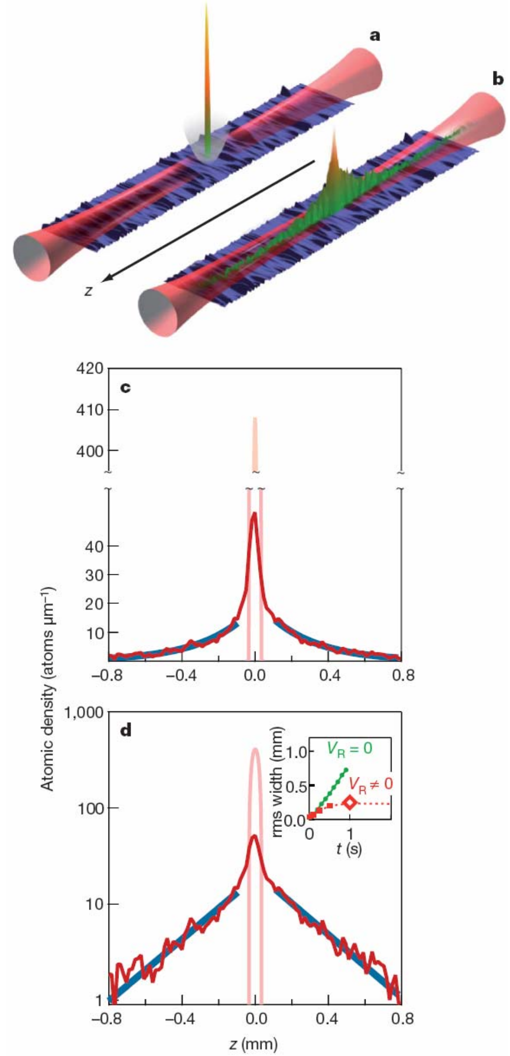
Outline

- The quartic Klein-Gordon (KG) disordered lattice
- Three different dynamical behaviors
- Numerical results
- Similarities with the disordered nonlinear Schrödinger equation (DNLS)
- Conclusions

Interplay of disorder and nonlinearity

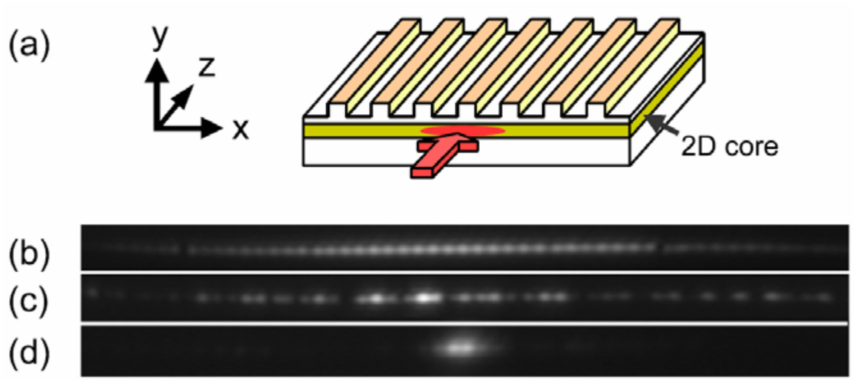
Waves in disordered media – Anderson localization (Anderson Phys. Rev. 1958). Experiments on BEC (Billy et al. Nature 2008)



Waves in nonlinear disordered media – localization or delocalization?

Theoretical and/or numerical studies (Shepelyansky PRL 1993, Molina Phys. Rev. B 1998, Pikovsky & Shepelyansky PRL 2008, Kopidakis et al. PRL 2008)

Experiments: propagation of light in disordered 1d waveguide lattices (Lahini et al. PRL 2008)



The Klein – Gordon (KG) model

$$H_K = \sum_{l=1}^{N} \frac{p_l^2}{2} + \frac{\tilde{\varepsilon}_l}{2} u_l^2 + \frac{1}{4} u_l^4 + \frac{1}{2W}\left(u_{l+1} - u_l\right)^2$$

with fixed boundary conditions $u_0=p_0=u_{N+1}=p_{N+1}=0$. Usually N=1000.

Parameters: W and the total energy E. $\tilde{\varepsilon}_l$ chosen uniformly from $\left[\frac{1}{2},\frac{3}{2}\right]$.

Linear case (neglecting the term $u_l^4/4$)

Ansatz: $u_l=A_l \exp(i\omega t)$

Eigenvalue problem: $\lambda A_l = \varepsilon_l A_l - (A_{l+1} + A_{l-1})$ with

$$\lambda = W\omega^2 - W - 2, \quad \varepsilon_l = W(\tilde{\varepsilon}_l - 1)$$

Unitary eigenvectors (normal modes - NMs) $A_{\nu,l}$ are ordered according to their center-of-norm coordinate: $X_\nu = \sum_{l=1}^{N} l A_{\nu,l}^2$

All eigenstates are localized (Anderson localization) having a localization length which is bounded from above.

Scales

$\omega_\nu^2 \in \left[\frac{1}{2}, \frac{3}{2} + \frac{4}{W}\right]$, width of the squared frequency spectrum: $\Delta_K = 1 + \frac{4}{W}$

Localization volume of eigenstate: $p_\nu = \frac{1}{\sum_{l=1}^{N} A_{\nu,l}^4}$

Average spacing of squared eigenfrequencies of NMs within the range of a localization volume: $\overline{\Delta\omega^2} = \frac{\Delta_K}{\overline{p_\nu}}$

For small values of W we have $\overline{\Delta\omega^2} \sim W^2$

Nonlinearity induced squared frequency shift of a single site oscillator

$$\delta_l = \frac{3E_l}{2\tilde{\varepsilon}_l} \propto E$$

The relation of the two scales $\overline{\Delta\omega^2} \le \Delta_K$ with the nonlinear frequency shift δ_l determines the packet evolution.

Distribution characterization

We consider normalized energy distributions in normal mode (NM) space

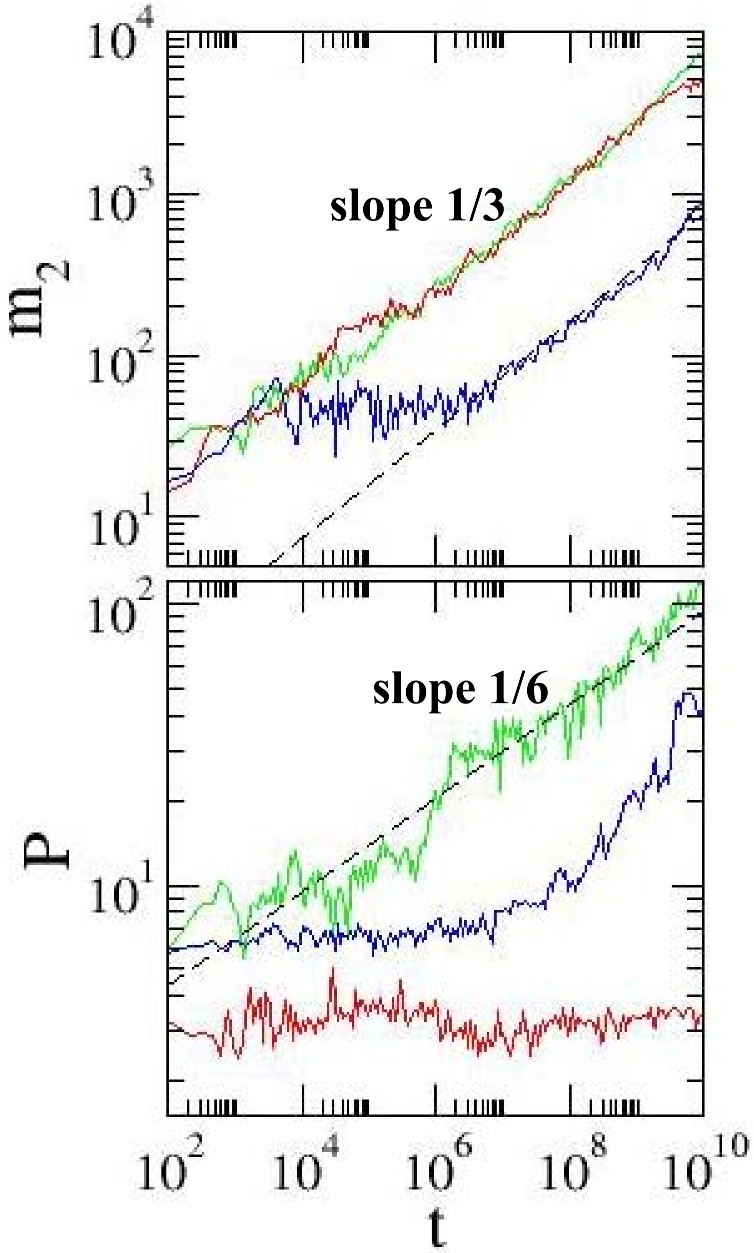
$$z_\nu \equiv \frac{E_\nu}{\sum_m E_m} \quad \text{with} \quad E_\nu = \frac{1}{2}\left(\dot{A}_\nu^2 + \omega_\nu^2 A_\nu^2\right)$$

, where A_ν is the amplitude of the νth NM.

Second moment: $$m_2 = \sum_{\nu=1}^{N} \left(\nu - \bar{\nu}\right)^2 z_\nu \quad \text{with} \quad \bar{\nu} = \sum_{\nu=1}^{N} \nu z_\nu$$

Participation number: $$P = \frac{1}{\sum_{\nu=1}^{N} z_\nu^2}$$

measures the number of stronger excited modes in z_ν. Single mode P=1, Equipartition of energy P=N.



E = 0.05, 0.4, 1.5 - W = 4. Single site excitations

Regime I: Small values of nonlinearity. $\delta_l < \overline{\Delta\omega^2}$ frequency shift is less than the average spacing of interacting modes. Localization as a transient (like in the linear case), with subsequent subdiffusion.

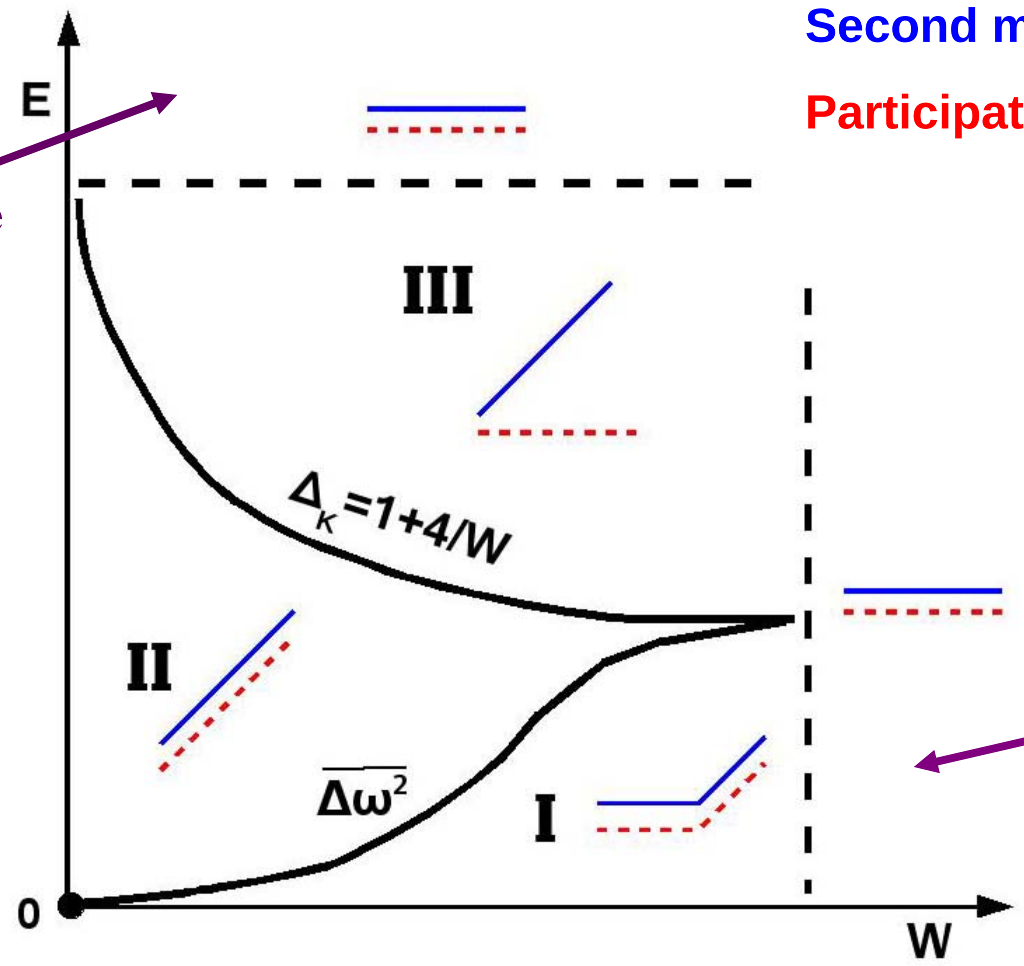
Regime II: Intermediate values of nonlinearity. $\Delta\omega^2 < \delta_l < \Delta_K$ resonance overlap may happen immediately. Immediate subdiffusion (Molina Phys. Rev. B 1998, Pikovsky & Shepelyansky PRL 2008).

Regime III: Big nonlinearities. $\delta_l > \Delta_K$ frequency shift exceeds the spectrum width. Some frequencies of NMs are tuned out of resonances with the NM spectrum, leading to selftrapping, while a small part of the wavepacket subdiffuses (Kopidakis et al. PRL 2008).

Subdiffusion: $m_2 \sim t^a, \quad P \sim t^{a/2}$

Assuming that the spreading is due to heating of the cold exterior, induced by the chaoticity of the wave packet, we theoretically predict α=1/3.

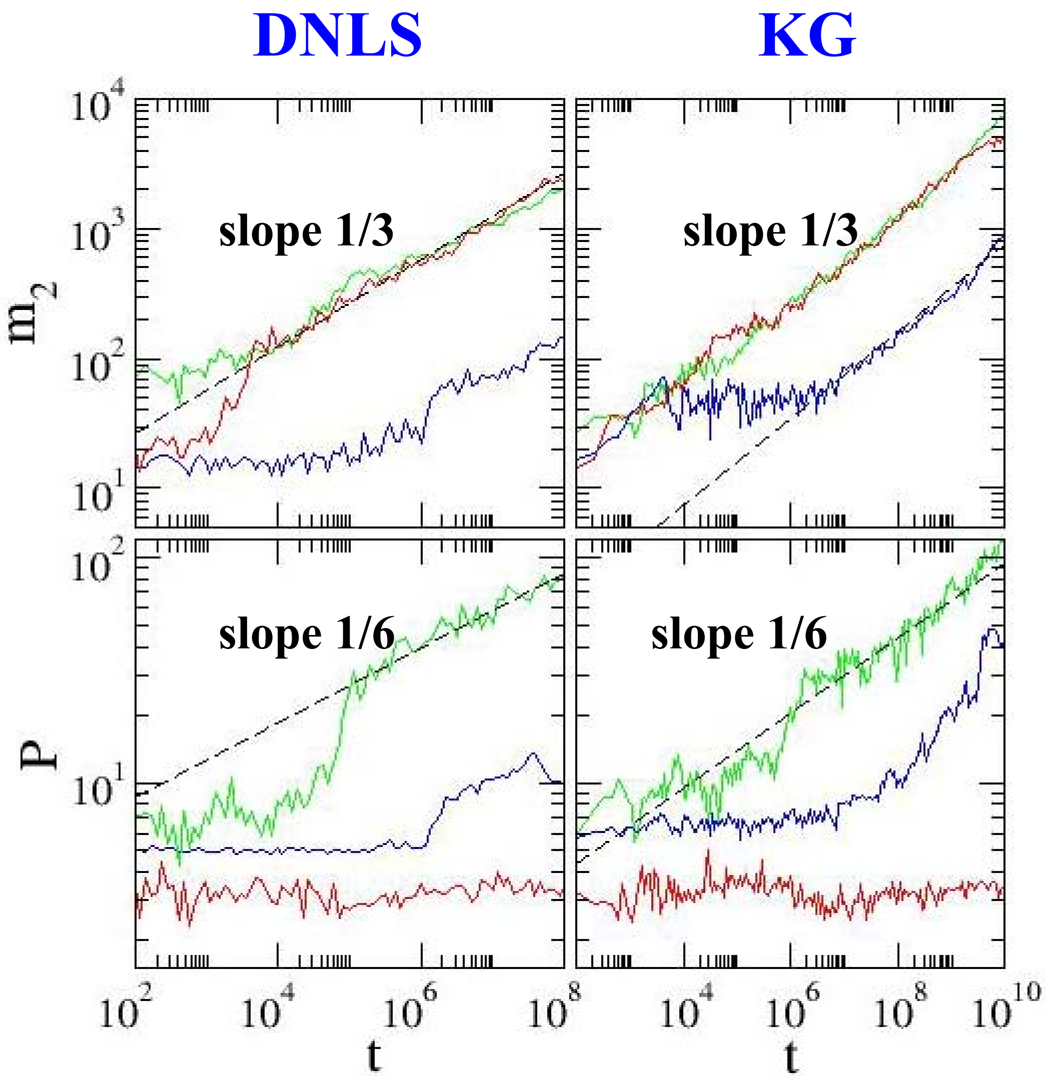
Different spreading regimes



The fraction of the wave packet that spreads decreases with increasing nonlinearity.

The detrapping time increases with increasing W.

Similar behavior of DNLS



Single site excitations

Regimes I, II, III

In regime II we averaged the measured exponent α over 20 realizations:

α=0.33±0.05 (KG)

α=0.33±0.02 (DLNS)

Conclusions

- Chart of different dynamical behaviors:
  - ✓ Weak nonlinearity: Anderson localization on finite times. After some detrapping time the wave packet delocalizes (Regime I)
  - ✓ Intermediate nonlinearity: wave packet delocalizes without transients (Regime II)
  - ✓ Strong nonlinearity: partial localization due to selftrapping, but a (small) part of the wave packet delocalizes (Regime III)
- Subdiffusive spreading induced by the chaoticity of the wavepacket
- Second moment of wavepacket ~ t^{α} with $\alpha=1/3$
- Spreading is universal due to nonintegrability and the exponent α does not depend on strength of nonlinearity and disorder

References

S. Flach, D.O. Krimer, ChS, 2009, PRL, 102, 024101.

ChS, D.O. Krimer, S. Komineas, S. Flach, 2009, arXiv:0901.4418